Density-density propagator for one-dimensional interacting spinless fermions with nonlinear dispersion and calculation of the Coulomb drag resistivity

A. V. Rozhkov
Institute for Theoretical and Applied Electrodynamics, ulica Izhorskaya 13/19, 124512 Moscow, Russian Federation
(Received 12 September 2007; revised manuscript received 9 January 2008; published 10 March 2008)

Using bosonization-fermionization transformation, we map the Tomonaga-Luttinger model of spinless fermions with nonlinear dispersion on the model of fermionic quasiparticles whose interaction is irrelevant in the renormalization group sense. Such mapping allows us to set up an expansion for the density-density propagator of the original Tomonaga-Luttinger Hamiltonian in orders of the (irrelevant) quasiparticle interaction. The lowest order term in such an expansion is proportional to the propagator for free fermions. The next term is also evaluated. The propagator found is used for calculation of the Coulomb drug resistivity r in a system of two capacitively coupled one-dimensional conductors. It is shown that r is proportional to T^2 for both free and interacting fermions. The marginal repulsive in-chain interaction acts to reduce r as compared to the noninteracting result. The correction to r due to the quasiparticle interaction is found as well. It scales as T^4 at low temperature.

DOI: 10.1103/PhysRevB.77.125109 PACS number(s): 71.10.Pm, 73.21.Hb

I. INTRODUCTION

The bosonization has been an indispensable tool for one-dimensional (1D) interacting fermion studies. The advantage of the bosonization is that it allows us to treat exactly the fermion interaction operator. This operator is marginal in the renormalization group (RG) sense and therefore cannot be dealt with the help of perturbative approximations.

However, the bosonization becomes inconvenient when one needs to go beyond the marginal operators. For example, when calculating the Coulomb drag resistivity in the system of two 1D wires,[1–4] it is necessary to account for the dispersion curvature v_F' of the fermions[5] which is an irrelevant operator. In the language of the bosonization, such operator introduces interaction between the bosons with the coupling constant v_F'. This destroys the solubility of the bosonized Hamiltonian.

Moreover, the Coulomb drag resistivity r is nonanalytical in v_F',

$$r \propto |v_F'|. \tag{1}$$

This indicates that simple perturbation theory in orders of v_F' is not applicable.

Different methods were used to address the issue of the nonlinear dispersion.[5–14] Unfortunately, these papers either rely on numerical calculation or exact solubility or employ uncontrollable approximations or devise methods suitable for a particular task at hand. No universal approach emerged from those works.

A detour around the bosonization was proposed in Refs. 15 and 16, where it was shown that a generic Tomonaga-Luttinger (TL) model of 1D interacting spinless fermions may be mapped on a system of free fermionic quasiparticles with weak irrelevant (in RG sense) interactions. The latter approach was particularly convenient for evaluation of the density-density correlation function. It was demonstrated that this correlation function is proportional to the density-density correlation function of the free fermions plus small corrections due to the interactions between the quasiparticles. In Refs. 15 and 16, the density-density retarded propagator $D_{k\omega}$ and the density spectral function $B_{k\omega}=-2\,\mathrm{Im}\,D_{k\omega}$ were determined to zeroth order in the quasiparticle interaction.

Since the Coulomb drag resistivity is a functional of $B_{k\omega}$, it is natural to apply the method of Refs. 15 and 16 to the problem of calculating the Coulomb drag resistivity. This is the purpose of this paper. More specifically, we will show that at small temperature, $r \approx aT^2$, where the coefficient a is a decreasing function of the in-chain repulsion, and that correction due to the quasiparticle interactions δr vanishes quicker than T^2: $\delta r \propto T^4$. These are two main results derived below.

The presentation is structured as follows. First, in Sec. II, we establish the mapping of the TL model on the quasiparticle model. Next, in Sec. III, we obtain the density-density propagator and the Coulomb drag resistivity to zeroth and first orders in the quasiparticle interaction. The conclusions of the paper are given in Sec. IV.

II. MAPPING

In this paper, we will study the model of 1D fermions whose Hamiltonian is

$$H = H_{\mathrm{kin}} + H_{\mathrm{nl}} + H_{\mathrm{int}}, \tag{2}$$

$$H_{\mathrm{kin}} = iv_F \int dx (:\psi_L^\dagger \nabla \psi_L: - :\psi_R^\dagger \nabla \psi_R:), \tag{3}$$

$$H_{\mathrm{nl}} = v_F' \int dx [:(\nabla\psi_L^\dagger)(\nabla\psi_L): + :(\nabla\psi_R^\dagger)(\nabla\psi_R):], \tag{4}$$

$$H_{\mathrm{int}} = g \int dx \rho_R \rho_L, \tag{5}$$

where $\psi_{R,L}$ are chiral fermionic fields corresponding to the right-moving (subscript R) and left-moving (subscript L) fermions, $\rho_{R,L}=:\psi_{R,L}^\dagger \psi_{R,L}:$ are chiral fermion densities, and the

colons denote normal ordering. The cutoff Λ is assumed for this quantum field theory.

A nonperturbative approach to handle Hamiltonian H was proposed in Ref. 15. There, a unitary operator U was constructed which transforms H into the quasiparticle Hamiltonian,

$$H_{\rm qp} = \tilde{H}_{\rm kin} + \tilde{H}_{\rm nl} + \tilde{H}'_{\rm int} + \delta\mu(N_R + N_L), \tag{6}$$

where $N_{R,L}$ are the total number of right-moving (left-moving) fermions, $\tilde{H}_{\rm kin}$ and $\tilde{H}_{\rm nl}$ have the same form as $H_{\rm kin}$ and $H_{\rm nl}$ but with $\tilde{v}_F$ and $\tilde{v}'_F$ instead of v_F and v'_F,

$$\tilde{H}_{\rm kin} = i\tilde{v}_F \int dx(:\tilde{\psi}^\dagger_L \nabla \tilde{\psi}_L: - :\tilde{\psi}^\dagger_R \nabla \tilde{\psi}_R:), \tag{7}$$

$$\tilde{H}_{\rm nl} = \tilde{v}'_F \int dx[:(\nabla\tilde{\psi}^\dagger_L)(\nabla\tilde{\psi}_L): + :(\nabla\tilde{\psi}^\dagger_R)(\nabla\tilde{\psi}_R):], \tag{8}$$

$$\tilde{v}_F = v_F\sqrt{1 - \left(\frac{g}{2\pi v_F}\right)^2}, \tag{9}$$

$$\tilde{v}'_F = \frac{v'_F}{4}(\mathcal{K}^{3/2} + 3\mathcal{K}^{-1/2}). \tag{10}$$

The usual TL parameter $\mathcal{K}$ is used in the last formula, $\mathcal{K} = \sqrt{(2\pi v_F - g)/(2\pi v_F + g)}$. Operators with tildes correspond to the quasiparticles: $\tilde{\psi}_p$ is the quasiparticle field, below we will use $\tilde{\rho}_p$, which is the quasiparticle density.

The quasiparticle interaction $\tilde{H}'_{\rm int}$ in Eq. (6) is given by the following expression:

$$\tilde{H}'_{\rm int} = -\sum_p ip\tilde{g}' \int dx\tilde{\rho}_{-p}[:\tilde{\psi}^\dagger_p(\nabla\tilde{\psi}_p): - :(\nabla\tilde{\psi}^\dagger_p)\tilde{\psi}_p:], \tag{11}$$

$$\tilde{g}' = \frac{\pi v'_F}{2}(\mathcal{K}^{3/2} - \mathcal{K}^{-1/2}). \tag{12}$$

In Eq. (11), the summation runs over the chirality index $p = R, L$ whose numerical values are $p = +1$ for R and $p = -1$ for L. Observe that the operator $\tilde{H}'_{\rm int}$ is irrelevant: its scaling dimension is equal to 3 which is greater than 2.[17] (To evaluate the scaling dimension, one has to add together the scaling dimensions of two fermion operators, the gradient and the electron density operator: $1/2 + 1/2 + 1 + 1 = 3$).

The chemical potential shift $\delta\mu$ in Eq. (6) is also induced by the transformation U. However, since such shift causes nothing but additional renormalization of the quasiparticle dispersion parameters $\tilde{v}_F$ and $\tilde{v}'_F$, we will not keep track of it below.

The readers who are interested to learn how operator U is constructed should consult Refs. 15 and 16. Here, we will use bosonization-fermionization sequence to establish the desired equivalence between H and $H_{\rm qp}$. That way, we will derive the result we need with no reference to the new technique unfamiliar to the majority of the researchers in the field.

The bosonization prescription[17,18] expresses the 1D chiral fermion field as an exponential of the bose field Φ and its conjugate Θ,

$$\psi^\dagger_p(x) = (2\pi a)^{-1/2}\eta_p e^{i\sqrt{\pi}[\Theta(x) + p\Phi(x)]}, \tag{13}$$

where $a \propto 1/\Lambda$ and η_p is the Klein factor. Consequences of this formula are

$$\rho_p = \frac{1}{2\sqrt{\pi}}(\nabla\Phi + p\nabla\Theta), \tag{14}$$

$$-ip:\psi^\dagger_p(\nabla\psi_p): + \text{H.c.} = \frac{1}{2}:(\nabla\Phi + p\nabla\Theta)^2:, \tag{15}$$

$$:(\nabla\psi^\dagger_p)(\nabla\psi_p): - \frac{1}{6}\nabla^2\rho_p = \frac{\sqrt{\pi}}{6}:(\nabla\Phi + p\nabla\Theta)^3:. \tag{16}$$

Using the above formulas, we can write the bosonic form of H,

$$H[\Phi,\Theta] = H_{\rm kin}[\Phi,\Theta] + H_{\rm int}[\Phi,\Theta] + H_{\rm nl}[\Phi,\Theta], \tag{17}$$

$$H_{\rm kin} + H_{\rm int} = \frac{\tilde{v}_F}{2}\int dx[\mathcal{K}:(\nabla\Theta)^2: + \mathcal{K}^{-1}:(\nabla\Phi)^2:], \tag{18}$$

$$H_{\rm nl} = \frac{\sqrt{\pi}}{6}v'_F\int dx\sum_p :(\nabla\Phi + p\nabla\Theta)^3:. \tag{19}$$

Equation (19) was derived in Ref. 19.

Once the bosonic form is explicitly written, we are ready for the second step of the derivation—rescaling of the bosonic fields: $\tilde{\Phi} = \mathcal{K}^{-1/2}\Phi$ and $\tilde{\Theta} = \mathcal{K}^{1/2}\Theta$. The different pieces of Eq. (17) can be expressed in terms of this boson as such

$$H_{\rm kin} + H_{\rm int} = \frac{\tilde{v}_F}{2}\int dx[:(\nabla\tilde{\Theta})^2: + :(\nabla\tilde{\Phi})^2:], \tag{20}$$

$$H_{\rm nl} = \frac{\sqrt{\pi}}{6}v'_F\int dx\sum_p :(\mathcal{K}^{1/2}\nabla\tilde{\Phi} + p\mathcal{K}^{-1/2}\nabla\tilde{\Theta})^3:. \tag{21}$$

Introducing a fermion (the quasiparticle) with the help of the formula

$$\tilde{\psi}^\dagger_p(x) = (2\pi a)^{-1/2}\eta_p e^{i\sqrt{\pi}[\tilde{\Theta}(x) + p\tilde{\Phi}(x)]}, \tag{22}$$

one can refermionize H. Namely, inverting Eq. (15), we obtain for the sum $H_{\rm kin} + H_{\rm int}$,

$$H_{\rm kin} + H_{\rm int} = \tilde{H}_{\rm kin}, \tag{23}$$

where $\tilde{H}_{\rm kin}$ is given by Eq. (7). On the right-hand side of this expression, the interaction term $\int dx\tilde{\rho}_R\tilde{\rho}_L$ is absent. Thus, the marginal interaction, the most troublesome part of the Hamiltonian, is removed.

The price we have to pay for the absence of the marginal interaction is that $H_{\rm nl}$ expressed in $\tilde{\Phi}$, $\tilde{\Theta}$ [Eq. (21)] cannot be easily fermionized. It is convenient to rewrite the latter equation,

$$H_{\rm nl} = \frac{\sqrt{\pi}}{6}\tilde{v}'_F \int dx \sum_p :(\nabla\tilde{\Phi} + p\,\nabla\,\tilde{\Theta})^3: + \frac{\tilde{g}'}{4\sqrt{\pi}} \int dx \sum_p :(\nabla\tilde{\Phi} + p\,\nabla\,\tilde{\Theta})^2(\nabla\tilde{\Phi} - p\,\nabla\,\tilde{\Theta}):. \quad (24)$$

The advantage of this form is that it depends on $(\nabla\tilde{\Phi} \pm \nabla\tilde{\Theta})$ combinations only. Thus, Eqs. (14)–(16) may be immediately applied and $H_{\rm nl}$ may be fermionized,

$$H_{\rm nl} = \tilde{H}_{\rm nl} + \tilde{H}'_{\rm int}, \quad (25)$$

where $\tilde{H}_{\rm nl}$ and $\tilde{H}'_{\rm int}$ are given by Eqs. (8) and (11).

This almost concludes the derivation of Eq. (6). What we lack is the $\delta\mu$ term of $H_{\rm qp}$. To obtain this term, we must (i) handle normal ordered expressions more accurately and (ii) take special care about the zero modes $N_{R,L}$. Technically, this is similar to the treatment of Refs. 15 and 16. Since we are not interested in the chemical potential shift, we will not address this issue here.

III. DENSITY-DENSITY PROPAGATOR AND THE COULOMB DRAG RESISTIVITY

Once the mapping of H on $H_{\rm qp}$ is established, we can use it to calculate the density-density propagator $D_{k\omega}$. First, let us find the following density-density correlation function,

$$\mathcal{R} = \langle[\rho_R(x,\tau) + \rho_L(x,\tau)][\rho_R(0,0) + \rho_L(0,0)]\rangle. \quad (26)$$

In the bosonic form, it is equal to

$$\mathcal{R} = \frac{1}{\pi}\langle\nabla\Phi(x)e^{-\tau H}\,\nabla\,\Phi(0)\rangle. \quad (27)$$

After the field rescaling, it becomes

$$\mathcal{R} = \frac{\mathcal{K}}{\pi}\langle\nabla\tilde{\Phi}(x)e^{-\tau H_{\rm qp}}\,\nabla\,\tilde{\Phi}(0)\rangle. \quad (28)$$

Under fermionization, $\mathcal{R}$ transforms into

$$\mathcal{R} = \mathcal{K}\langle[\tilde{\rho}_R(x,\tau) + \tilde{\rho}_L(x,\tau)][\tilde{\rho}_R(0,0) + \tilde{\rho}_L(0,0)]\rangle_{\rm qp}, \quad (29)$$

where the subscript "qp" reminds that the averaging is to be performed with respect to the quasiparticle Hamiltonian $H_{\rm qp}$.

One can prove through the same line of reasoning that

$$D_{k\omega} = \mathcal{K}\tilde{D}_{k\omega}, \quad (30)$$

where $\tilde{D}_{k\omega}$ is the density-density propagator for the quasiparticle Hamiltonian $H_{\rm qp}$. Consequently, the task of finding the density-density propagator for the physical fermions is reduced to the task of finding the quasiparticle density-density propagator. The latter is much easier for the quasiparticle interaction, $\tilde{H}'_{\rm int}$ is irrelevant in the RG sense.

As a starting point, we calculate $D_{k\omega}$ to zeroth order in $\tilde{H}'_{\rm int}$,[15,16]

$$D^0_{k\omega} = \frac{\mathcal{K}}{4\pi\tilde{v}'_F k}\ln\left[\frac{(\tilde{v}_F k - \tilde{v}'_F k^2)^2 - (\omega + i0)^2}{(\tilde{v}_F k + \tilde{v}'_F k^2)^2 - (\omega + i0)^2}\right]. \quad (31)$$

The superscript "0" in the above formula indicates that the expression is valid to zeroth order in $\tilde{g}'$.

The spectral function is therefore,

$$B^0_{k\omega} = \frac{\mathcal{K}}{2\tilde{v}'_F k}[\vartheta(\omega^2 - (\tilde{v}_F k - \tilde{v}'_F k^2)^2) - \vartheta(\omega^2 - (\tilde{v}_F k + \tilde{v}'_F k^2)^2)]\,\mathrm{sgn}\,\omega. \quad (32)$$

This expression can be further used for evaluation of the Coulomb drag resistivity. Before proceeding with such calculations, let us briefly explain what is Coulomb drag.

In the Coulomb drag experiment, two parallel 1D wires (subscript $i=1,2$) of length L with Hamiltonians H [Eq. (2)] are coupled capacitively with the Hamiltonian $H_C = g_C\int dx\rho_1\rho_2$. Because of this coupling, electrical current I in one of the wires induces potential drop V across the other wire. The proportionality coefficient between V and I is called the Coulomb drag resistivity $r=V/IL$. It characterizes the pulling force which the fermions in the current-carrying wire exert on the fermions in the other wire. The experimental physics of the Coulomb drag is quite rich: the observed values of r could be either positive or negative and show dependence on temperature, spacial inhomogeneity, and applied magnetic field.

In general, two mechanisms for the Coulomb drag are discussed in the theoretical literature.[3] According to one mechanism, the drag occurs because the Wigner crystallike correlations in both wires lock against each other. In the RG language, this corresponds to the relevance of the interwire backscattering interaction. Such mechanism works best at zero temperature but quickly deteriorates at $T>0$. We will not study this mechanism in our paper.

The second mechanism is insensitive to the interwire backscattering. Instead, it relies upon interaction of the smooth components of the electron densities. It is more resilient toward temperature, but the resultant value of r is proportional to $|v'_F|$. Because of this, it is impossible to study the second mechanism within the well-established framework of the one-dimensional bosonization. The purpose of this paper is to provide a reliable approach overcoming this difficulty.

We start with the following formula for r [Eq. (7) of Ref. 5] which is valid when the interwire backscattering can be neglected,

$$r = \frac{g_C^2}{16\pi^3 n^2 T}\int_0^{+\infty} dk \int_0^{+\infty} d\omega \frac{k^2 B_{1,k\omega}B_{2,k\omega}}{\sinh^2(\omega/2T)}. \quad (33)$$

Here, n is the electron density. The spectral density $A(k,\omega)$ of Ref. 5 relates to our spectral density $B_{k\omega}$ as $B_{k\omega} = -2A(k,\omega)$. In that reference, the notation U_{12} is used for the interwire coupling constant g_C. Below, we will study the case of identical wires: $B_1 = B_2 = B$.

Since we know $B_{k\omega}$ to zeroth order in $\tilde{g}'$, it is straightforward to find r with the same accuracy. Substituting $B^0_{k\omega}$ into Eq. (33), we obtain

$$r = \frac{g_C^2 \mathcal{K}^2}{64\pi^3 (\tilde{v}_F')^2 n^2 T} \int_0^{+\infty} dk \int_{\omega_-}^{\omega_+} d\omega \frac{1}{\sinh^2(\omega/2T)}, \quad (34)$$

$$\omega_\pm = \tilde{v}_F k \pm |\tilde{v}_F'| k^2. \quad (35)$$

Evaluating the integral,

$$\int_{\omega_-}^{\omega_+} d\omega \frac{1}{\sinh^2(\omega/2T)} \approx \frac{2|\tilde{v}_F'| k^2}{\sinh^2(\tilde{v}_F k/2T)}, \quad (36)$$

we find

$$r = \frac{g_C^2 \mathcal{K}^2}{24\pi \tilde{v}_F^3 |\tilde{v}_F'| n^2} T^2, \quad (37)$$

where we used the identity

$$\int_0^{+\infty} \frac{x^2 dx}{\sinh^2 x} = \frac{\pi^2}{6}. \quad (38)$$

The electron density, which enters Eq. (37), is not an independent quantity. It can be expressed in terms of the dispersion parameters: $n = v_F/(2\pi v_F')$.

Furthermore, it is possible to show with the help of Eqs. (9) and (10) that $v_F = \tilde{v}_F + O(g^2)$ and $v_F' = \tilde{v}_F' + O(g^2)$. Since our accuracy does not allow us to keep $O(g^2)$ terms, we can replace $\tilde{v}_F$ by v_F and $\tilde{v}_F'$ by v_F' in Eq. (37). Thus, it is true that

$$r \approx aT^2, \quad (39)$$

$$a = \frac{\pi g_C^2 \mathcal{K}^2 |v_F'|}{6 v_F^5} + O(g^2). \quad (40)$$

The interaction enters the expression for a through $\mathcal{K}^2 = 1 - g/(\pi v_F) + O(g^2)$. We see that the repulsive in-chain interaction ($g > 0$) acts to reduce r.

Equation (39) may be cast in the following form:

$$r \approx \frac{c_1 \mathcal{K}^2}{l_0} \left(\frac{T}{\epsilon_F} \right)^2, \quad (41)$$

where l_0^{-1} and ϵ_F are defined in Ref. 5. They are

$$\frac{1}{l_0} = \left(\frac{g_C}{2\pi v_F} \right)^2 n = \frac{g_C^2}{8\pi^3 v_F' v_F}, \quad (42)$$

$$\epsilon_F = v_F' k_F^2 = \frac{v_F^2}{4 v_F'}. \quad (43)$$

The numerical coefficient c_1 is equal to $\pi^4/12$. For free fermions ($\mathcal{K} = 1$), the formula similar to Eq. (41) was derived in Ref. 5.

Furthermore, that paper established that $r \propto T^2$ for both noninteracting fermions and for exactly soluble Calogero-Sutherland model. Our Eq. (37) proves that T^2 dependence holds for a generic interacting model as well.

Despite general agreement between our Eq. (41) and Eq. (13) of Ref. 5, a discrepancy does exists. In Ref. 5, two values for c_1 were reported [see discussion after Eq. (13) in the latter reference]: $\pi^4/12$ for approximate spectral function and $\pi^2/4$ for exact spectral function. Why our c_1 coincides with the approximate c_1 of Ref. 5 and not with the exact is unclear.

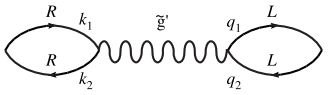

FIG. 1. Feynman diagram corresponding to the lowest order interaction correction to the density-density propagator.

Equation (39) accounts for effects of marginal interaction g. In addition to that, our method allows us to evaluate the correction δr to Eq. (39) due to the quasiparticle interaction. To find δr, we calculate the lowest order correction to $D_{k\omega}^0$. Such correction appears in the first order in $\tilde{g}'$: since $\tilde{H}_{\rm int}'$ couples the right-moving and the left-moving quasiparticles, the expectation value $\langle \tilde{\rho}_R(x,\tau)\tilde{\rho}_L(0,0)\rangle$ taken with respect to $H_{\rm qp}$ [Eq. (6)] is no longer zero but instead $O(\tilde{g}')$. Thus, full Matsubara propagator $\mathcal{D} = \mathcal{D}^0 + \delta\mathcal{D}$, where

$$\delta\mathcal{D}(x,\tau) = \mathcal{K}\delta\tilde{\mathcal{D}}(x,\tau) \approx \mathcal{K} \int d\tau' [\langle\langle \tilde{\rho}_R(x,\tau)\tilde{H}_{\rm int}'(\tau')\tilde{\rho}_L(0,0)\rangle\rangle_0 + \langle\langle \tilde{\rho}_L(x,\tau)\tilde{H}_{\rm int}'(\tau')\tilde{\rho}_R(0,0)\rangle\rangle_0], \quad (44)$$

where the symbol $\langle\langle \cdots \rangle\rangle_0$ stands for time-ordered averaging with respect to the noninteracting ($\tilde{g}' = 0$) quasiparticle Hamiltonian.

The Feynman diagram which describes $\delta\tilde{\mathcal{D}}$ is shown on Fig. 1. The wavy interaction line is to be identified with $\tilde{g}'(k_1 + k_2 - q_1 - q_2)$, k_i, q_i are the fermion momenta.

When evaluating $\delta\mathcal{D}$, we note that in the noninteracting quasiparticle Hamiltonian, the left-moving and the right-moving quasiparticles are decoupled from each other. Consequently, the object $\langle\langle \rho_{L,R} H_{\rm int}' \rho_{L,R}\rangle\rangle_0$ is split into products of left-only and right-only expectation values. As a result, one can show that

$$\delta\mathcal{D}_{k\omega} = -2\tilde{g}'\mathcal{K} \sum_p \tilde{\mathcal{D}}_{pk\omega}^0 \tilde{\mathcal{P}}_{-pk\omega}, \quad (45)$$

where the chiral noninteracting propagator is

$$\tilde{\mathcal{D}}_{pk\omega}^0 = \frac{p}{4\pi \tilde{v}_F' k} \ln\left(\frac{i\omega - p\tilde{v}_F k + \tilde{v}_F' k^2}{i\omega - p\tilde{v}_F k - \tilde{v}_F' k^2} \right), \quad (46)$$

and the propagator $\tilde{\mathcal{P}}_p$ is defined as

$$\tilde{\mathcal{P}}_p(x,\tau) = -ip\langle\langle \tilde{\rho}_p(x,\tau)\{:\tilde{\psi}_p^\dagger(0,0)[\nabla\tilde{\psi}_p(0,0)]: - :[\nabla\tilde{\psi}_p^\dagger(0,0)]\tilde{\psi}_p(0,0):\}\rangle\rangle_0. \quad (47)$$

It equals

$$\tilde{\mathcal{P}}_{pk\omega} = -\frac{1}{2\pi\tilde{v}_F'} + \frac{(i\omega - p\tilde{v}_F k)}{p\tilde{v}_F' k} \tilde{\mathcal{D}}_{pk\omega}^0. \quad (48)$$

To find the correction to the spectral function δB, it is necessary to perform the analytic continuation in Eq. (45): $\delta D_{k\omega} = -2\tilde{g}'\mathcal{K}\Sigma_p \tilde{D}_{-pk\omega}^0 \tilde{P}_{pk\omega}$, where the retarded quantity $\tilde{P}_{pk\omega} = \tilde{\mathcal{P}}_{pk\omega}|_{i\omega\to\omega+i0}$ is equal to

$$\tilde{P}_{pk\omega} = -\frac{1}{2\pi\tilde{v}'_F} + \frac{(\omega - p\tilde{v}_F k + i0)}{p\tilde{v}'_F k}\tilde{D}^0_{pk\omega}. \tag{49}$$

Introducing the chiral spectral density, $\tilde{B}^0_p = -2\,\mathrm{Im}\,\tilde{D}^0_p$, we obtain for δB,

$$\delta B_{k\omega} = \frac{\tilde{g}'\mathcal{K}}{\pi\tilde{v}'_F}\sum_p \tilde{B}^0_{-pk\omega}(4\pi\tilde{v}_F\,\mathrm{Re}\,\tilde{D}^0_{pk\omega} + 1). \tag{50}$$

Observe that $\tilde{B}^0_{-p}$ is nonzero only in a narrow region $|\omega + p\tilde{v}_F k| < |\tilde{v}'_F|k^2$. Thus, defining $\delta\omega_p = \omega + p\tilde{v}_F k$ [$\delta\omega_p = O(k^2)$], one can expand $\mathrm{Re}\,\tilde{D}^0_p$ in orders of k,

$$\delta B_{k\omega} \approx \frac{\tilde{g}'\mathcal{K}}{\pi\tilde{v}'_F}\sum_p \tilde{B}^0_{-pk\omega}\left[-\frac{p\,\delta\omega_p}{2\tilde{v}_F k} - \frac{(\delta\omega_p)^2}{4\tilde{v}_F^2 k^2} - \frac{(\tilde{v}'_F k)^2}{12\tilde{v}_F^2}\right]. \tag{51}$$

Placing $B_{k\omega} \approx B^0_{k\omega} + \delta B_{k\omega}$ into Eq. (33), one finds

$$\delta r \propto T^4. \tag{52}$$

Consequently, at low temperature, the interaction correction δr to the Coulomb drag resistivity r vanishes quicker than T^2. Thus, the noninteracting quasiparticle result [Eq. (37)] suffices to capture the leading behaviour of r at $T \to 0$.

IV. CONCLUSIONS

Using bosonization-fermionization trick, we mapped the Tomonaga-Luttinger Hamiltonian with nonlinear dispersion on the Hamiltonian of the quasiparticles with irrelevant interaction. This mapping allows us to evaluate the density-density propagator of the Tomonaga-Luttinger model with nonlinear dispersion. The propagator itself was used to calculate the temperature dependence of the Coulomb drag resistivity r. It was established that $r \propto T^2$ at low T for both interacting and free fermions. The irrelevant quasiparticle interaction introduces additional correction which vanishes as T^4.

ACKNOWLEDGMENTS

The author is grateful for the support provided by the Dynasty Foundation and by the RFBR under Grants No. 06-02-16691 and 06-02-91200.

[1] P. Debray, V. Zverev, O. Raichev, R. Klesse, P. Vasilopoulos, and R. S. Newrock, J. Phys.: Condens. Matter **13**, 3389 (2001).
[2] M. Yamamoto, M. Stopa, Y. Tokura, Y. Hirayama, and S. Tarucha, Physica E (Amsterdam) **12**, 726 (2002).
[3] P. Debray, V. N. Zverev, V. Gurevich, R. Klesse, and R. S. Newrock, Semicond. Sci. Technol. **17**, R21 (2002).
[4] M. Yamamoto, M. Stopa, Y. Tokura, Y. Hirayama, and S. Tarucha, Science **313**, 204 (2006).
[5] M. Pustilnik, E. G. Mishchenko, L. I. Glazman, and A. V. Andreev, Phys. Rev. Lett. **91**, 126805 (2003).
[6] M. Pustilnik, M. Khodas, A. Kamenev, and L. I. Glazman, Phys. Rev. Lett. **96**, 196405 (2006).
[7] Peter Kopietz, *Bosonization of Interacting Fermions in Arbitrary Dimensions* (Springer, New York, 1997).
[8] Peyman Pirooznia and Peter Kopietz, Eur. Phys. J. D **58**, 291 (2007).
[9] K. V. Samokhin, J. Phys.: Condens. Matter **10**, L533 (1998).
[10] Sofian Teber, Eur. Phys. J. B **52**, 233 (2006).
[11] Sofian Teber, Phys. Rev. B **76**, 045309 (2007).
[12] Michael Pustilnik, Phys. Rev. Lett. **97**, 036404 (2006).
[13] D. N. Aristov, Phys. Rev. B **76**, 085327 (2007).
[14] R. G. Pereira, J. Sirker, J.-S. Caux, R. Hagemans, J. M. Maillet, S. R. White, and I. Affleck, J. Stat. Mech.: Theory Exp. (2007) P08022.
[15] A. V. Rozhkov, Eur. Phys. J. B **47**, 193 (2005).
[16] A. V. Rozhkov, Phys. Rev. B **74**, 245123 (2006).
[17] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, UK, 1998).
[18] T. Giamarchi, *Quantum Physics in One Dimension* (Clarendon, Oxford, UK, 2007).
[19] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).